The Equatorial Coordinate System

The conventions, and how to apply them

Everything in this system turns on two questions: where you measure from, and which way round you count. Get those right and nothing else can go wrong. This is the companion to Episode 3 — the rules in one place, with worked examples and the mistakes that are actually made.

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1. The two coordinates

A body's position on the celestial sphere is fixed by two angles.

	declination	**hour angle**
measured along	the body's hour circle	the celestial equator
measured from	the celestial equator	a chosen meridian
direction	north or south	**west, always**
range	0° to 90°	0° to 360°
named	N or S	no suffix at all

Note the asymmetry. Declination carries a name, as latitude does. An hour angle never does: it runs the whole way round, so there is nothing for a suffix to distinguish.

2. The counting directions

Three quantities are measured along the celestial equator, and the direction is not the same for all of them.

	from	**direction**	**units**
GHA	the Greenwich meridian	west	degrees
LHA	your own meridian	west	degrees
SHA	the First Point of Aries	west	degrees
RA (α)	the First Point of Aries	**east**	**hours**

Right ascension is the odd one out twice over: it is the only one counted eastward, and the only one given in time. Since the two run opposite ways from the same origin, they make a full turn between them:

$$\mathrm{SHA} = 360^\circ - \alpha, \qquad 1^{\mathrm{h}} = 15^\circ, \qquad 1^{\mathrm{m}} = 15', \qquad 1^{\mathrm{s}} = 15''.$$

Examples. $\alpha = 16^{\mathrm{h}}\,40^{\mathrm{m}} = 250^\circ\,00.0'$, so SHA $= 110^\circ\,00.0'$. $\alpha = 14^{\mathrm{h}}\,15.6^{\mathrm{m}} = 213^\circ\,54.0'$, so SHA $= 146^\circ\,06.0'$. $\alpha = 2^{\mathrm{h}}\,08.4^{\mathrm{m}} = 32^\circ\,06.0'$, so SHA $= 327^\circ\,54.0'$.

3. From GHA to LHA

This is the step where your own position enters the system. GHA belongs to the body and the instant and is the same for everyone on Earth; LHA is yours.

$$\boxed{\mathrm{LHA} = \mathrm{GHA} - \lambda_W \qquad\qquad \mathrm{LHA} = \mathrm{GHA} + \lambda_E}$$

The rule. West longitude subtracts, east longitude adds. If the result falls outside 0° to 360°, add or take off a whole turn — the convention insists on it, and a negative hour angle is not a thing.

Worked examples

GHA	longitude	working	LHA
$312^\circ\,14.0'$	$58^\circ\,22.0'$ W	subtract	$253^\circ\,52.0'$
$41^\circ\,06.0'$	$122^\circ\,48.0'$ W	subtract, then $+360^\circ$	$278^\circ\,18.0'$
$274^\circ\,52.0'$	$35^\circ\,10.0'$ E	add	$310^\circ\,02.0'$
$338^\circ\,30.0'$	$47^\circ\,55.0'$ E	add, then -360°	$026^\circ\,25.0'$

The second and fourth lines are the ones worth practising: both run off the end of the range, in opposite directions, and both are fixed by a single full turn.

4. A star, from the beginning

For a star the chain has one extra link, because the star's own hour angle is not tabulated directly.

$$\text{GHA star} = \text{GHA Aries} + \text{SHA star}$$

Worked example. Greenwich hour angle of Aries $118^\circ\,23.4'$; the star's SHA $80^\circ\,30.2'$; observer in $58^\circ\,22.0'$ W.

GHA Aries	$118^\circ\,23.4'$
SHA star	$+\,80^\circ\,30.2'$
GHA star	$198^\circ\,53.6'$
longitude W	$-\,58^\circ\,22.0'$
LHA star	$\mathbf{140^\circ\,31.6'}$

Declination is taken as it stands — it needs no such treatment, because it is not measured from a meridian at all.

5. Polar distance

The same arc as the declination, counted from the other end of the hour circle:

$$p = 90^\circ - \text{Dec} \qquad \text{(same name as the elevated pole)}$$
$$p = 90^\circ + \text{Dec} \qquad \text{(contrary name)}$$

It is one side of the navigational triangle — the side from the pole to the body — which is why it is worth holding on to. The contrary-name case needs an observer and an elevated pole to make sense of, and belongs with that triangle.

6. Checks worth carrying

LHA always increases. The counting runs the same way the sky turns, so as time passes LHA grows. If yours is running backwards, you have added a west longitude or subtracted an east one.

LHA zero means the meridian. The body is due north or due south of you, on your own meridian. LHA 180° puts it on the opposite one.

Declination has a name, hour angles do not. If you have written "LHA 046° W" the suffix is meaningless — hour angles are already counted one way only.

Minutes, not decimals of a degree. 118° 23.4′ is one hundred and eighteen degrees and twenty-three point four *minutes*, not 118.234°. The commonest arithmetic slip in the whole subject.

A minute of declination is a mile. A minute of hour angle is not. Declination is measured along the body's hour circle, which is a great circle, so a minute of it is a minute of arc in full. An hour angle is an angle at the pole: measured along the equator it does subtend a mile per minute, but the distance it represents *at the body* lies along the body's parallel of declination, which is a small circle. There the same angle subtends cos(Dec) of a mile per minute, and at a declination of 60° that is half.

7. What this document does not cover

Deliberately left out, each being a subject of its own: **time** — the sidereal and solar day, the mean sun, the equation of time, and the conversion of arc to time; **the almanac** — its pages, increments and corrections; and **sight reduction** — the horizon system, altitude and azimuth, and the navigational triangle that joins the two systems together.

Every figure in this document has been computed rather than copied. The worked examples can be checked against each other: the chain in section 4 uses the same longitude as the first line of section 3.