Time in Astronavigation

Where the minutes go, and how a watch gives longitude

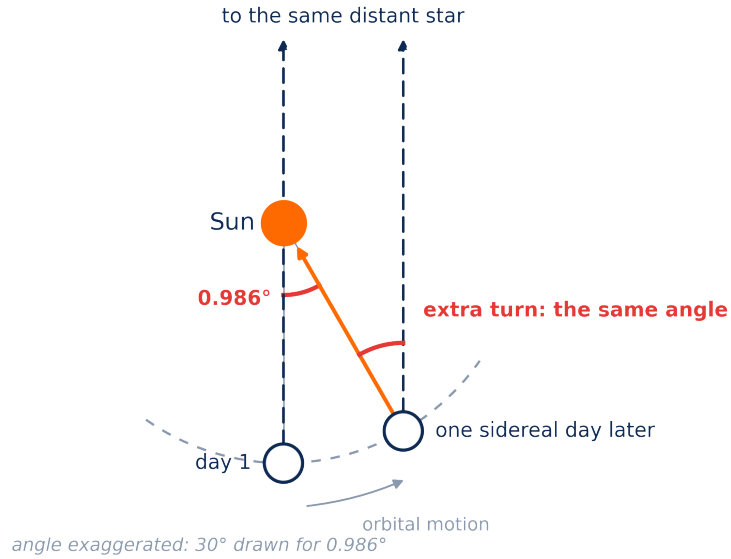
A day depends on what you measure it against, and the Sun is a poor clock. This is the companion to Episode 4 — where the missing four minutes come from, the equation of time across a whole year, the two kinds of year and of month, and a longitude from the noon Sun worked through step by step.

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1. Where the four minutes go

Turn relative to what? Against a distant star the Earth turns once in a **sidereal day**. Against the Sun it needs a little more, because in the meantime it has moved along its orbit, in the same direction as it spins.



Step 1. The Earth goes once round the Sun in a year, so each day it moves on by

$$\frac{360^\circ}{365.2422\ \mathrm{d}} = 0.9856^\circ \text{ a day.}$$

Step 2. The direction of the Sun swings by that same angle — the two red arcs in the figure are alternate angles between parallel lines. So in one mean solar day the Earth turns not 360° but 360.9856° against the stars.

Step 3. The extra 0.9856° is the part of the day spent turning beyond one full rotation:

$$\frac{0.9856^\circ}{360.9856^\circ} \times 24^\mathrm{h} = 235.9\ \mathrm{s} = 3^\mathrm{m}\,55.9^\mathrm{s} \approx 3^\mathrm{m}\,56^\mathrm{s}.$$

The quick version — 0.986° at four minutes a degree — gives $3^\mathrm{m}\,56.6^\mathrm{s}$; the small difference is that the Earth turns at 360.9856° per day, not 360°.

Result.

$$\boxed{\text{sidereal day} = 24^\mathrm{h} - 3^\mathrm{m}\,55.9^\mathrm{s} = 23^\mathrm{h}\,56^\mathrm{m}\,04.09^\mathrm{s}}$$

Strictly, the sidereal day is measured against the First Point of Aries, which drifts slowly westward through the stars (precession). The day measured against the fixed stars themselves, sometimes called the *stellar* day, is about 0.008 s longer. Nothing at sea depends on the difference.

2. Three bodies, three rates

Spread over a full turn of 360°, each body's hour angle grows at its own rate.

	per day	per hour	why
Aries	360.9856°	15° 02.46′	fixed among the stars
mean Sun	360°	15° 00.00′	the definition of the clock
Moon	347.81°	14° 29.5′	moves 13.18° a day eastward

The Moon. It goes round the stars in a sidereal month of 27.3217 days, so it moves eastward by $360^\circ/27.3217 = 13.18^\circ$ a day, against the sky's westward turn:

$$360.9856^\circ - 13.18^\circ = 347.81^\circ \text{ a day} \quad \Rightarrow \quad \frac{347.81^\circ}{24} = 14.492^\circ = 14^\circ\,29.5' \text{ an hour.}$$

To return to your meridian it must therefore turn $360^\circ/347.81^\circ = 1.0350$ days, which is $24^{\mathrm{h}}\,50.5^{\mathrm{m}}$. The same answer comes from the synodic month (section 4):

$$24^{\mathrm{h}} \times \frac{29.5306}{28.5306} - 24^{\mathrm{h}} = 50.5^{\mathrm{m}}.$$

So the Moon crosses your meridian about fifty minutes later each day.

A note for later. The Moon's rate is not constant: its orbit is elliptical and inclined, so its hourly rate changes from day to day. The tables navigators use are built on a fixed base of 14° 19.0′ an hour, and a separate correction adds the excess. 14° 29.5′ is the average — not a contradiction. The tables are an episode of their own.

3. Sidereal time

A clock that keeps the sidereal day keeps **sidereal time**. It is simply the hour angle of Aries, written in hours instead of degrees:

$$\boxed{\text{GHA Aries} = 15^\circ \times \text{GST} \qquad \text{LHA Aries} = 15^\circ \times \text{LST}}$$

with GST in hours, and LST obtained from GST exactly as LHA is obtained from GHA: west longitude subtracts, east adds.

Example. 3 November 2026, 13:04 UT. The standard expression for Greenwich sidereal time gives $\text{GST} = 15^{\mathrm{h}}\,55^{\mathrm{m}}\,13^{\mathrm{s}}$, so

$$\text{GHA Aries} = 15 \times 15^{\mathrm{h}}\,55^{\mathrm{m}}\,13^{\mathrm{s}} = 238^\circ\,48.3', \qquad \text{LHA Aries } (20^\circ\text{ W}) = 218^\circ\,48.3'.$$

A navigator never needs sidereal time by that name; it arrives already converted to degrees, as the hour angle of Aries.

4. Two kinds of year, two kinds of month

The same question — measured against what? — splits the year and the month as it split the day.

	measured against	length	used for
sidereal year	the fixed stars	365.2564 d	the orbit itself
tropical year	the First Point of Aries	365.2422 d	seasons, the calendar
sidereal month	the fixed stars	27.3217 d	the Moon's orbit
synodic month	the Sun	29.5306 d	phases, new Moon to new Moon

The year. The difference is 0.0142 d, about 20.4 minutes. The First Point of Aries moves westward by $50.3''$ a year to meet the Sun, so the Sun returns to it slightly before it returns to the same stars. That slow drift is precession. The calendar follows the tropical year, because the seasons do.

The month. The difference is 2.21 days. While the Moon goes once round the stars, the Sun has moved about 27° along the ecliptic, and the Moon needs two more days to catch it up and repeat the same phase.

One rule for all three. The sidereal day against the solar day, the sidereal month against the synodic month: in each pair the second is longer because the reference has moved on, by exactly one extra turn a year.

$$\frac{1}{P_{\text{sidereal}}} - \frac{1}{P_{\text{against the Sun}}} = \frac{1}{\text{year}} \qquad \text{e.g.} \quad \frac{1}{27.3217} - \frac{1}{29.5306} = \frac{1}{365.26}.$$

5. The mean Sun and the equation of time

The **mean Sun** moves along the celestial equator at a constant rate, completing its circuit in a tropical year. Clocks keep its time:

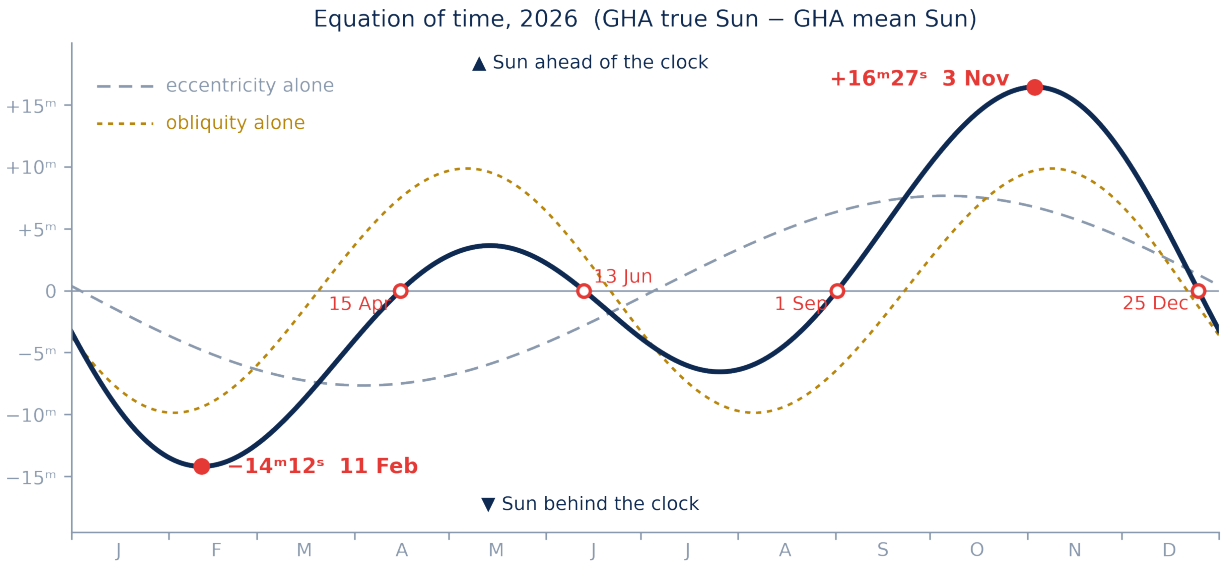
$$\text{GMT} = \text{GHA mean Sun} + 12^{\text{h}}, \qquad \text{LMT} = \text{LHA mean Sun} + 12^{\text{h}}.$$

The **equation of time** is the gap between the true Sun and the mean one:

$$\boxed{\text{EoT} = \text{GHA true Sun} - \text{GHA mean Sun}}$$

Since $\text{GHA} = \text{GHA Aries} - \text{RA}$ for any body, the same quantity can be written $\text{RA}_{\text{mean}} - \text{RA}_{\text{true}}$, and that is how it is computed.

The sign convention. Positive means the true Sun is ahead of the clock: it reaches the meridian before 12:00 mean time. This document and the film use that convention throughout. Some older British texts define the equation of time the other way round, as mean minus apparent, which flips every sign. Whatever table or book you use, check which convention it follows before you add or subtract.



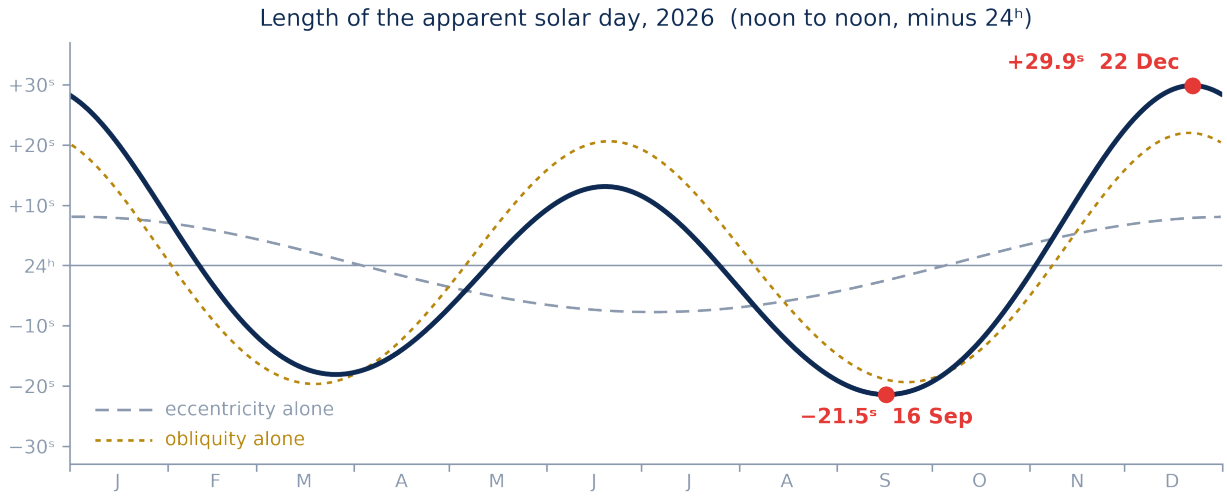
Two causes, added. The two dashed curves are the two reasons from the film, each computed on its own. The eccentricity of the orbit contributes a wave with one cycle a year and an amplitude of $7^{m}\,40^{s}$; the tilt of the ecliptic, a wave with two cycles a year and an amplitude of $9^{m}\,52^{s}$. The solid curve is their sum, which is why it has two unequal peaks and two unequal troughs.

date (2026)	EoT	Sun on the Greenwich meridian
11 February	$-14^{m}\,12^{s}$	12:14:12 UT
15 April	0	12:00
14 May	$+3^{m}\,38^{s}$	11:56
13 June	0	12:00
26 July	$-6^{m}\,34^{s}$	12:07
1 September	0	12:00
3 November	$+16^{m}\,27^{s}$	11:43:33 UT
25 December	0	12:00

The meridian passage at Greenwich is simply 12^{h} − EoT: about 12:14 in mid-February and 11:44 at the start of November — the two times in the opening of the film. The values are for 2026; from year to year they move by seconds, and the dates of the zeros by a day at most.

6. The length of the apparent solar day

Measure from one Greenwich noon to the next. The interval is 24 hours minus the day's change in the equation of time, and it wanders through the year.



	longest	shortest
both effects (2026)	$24^{h}\,00^{m}\,29.9^{s}$, from 22 Dec	$23^{h}\,59^{m}\,38.5^{s}$, from 16 Sep
eccentricity alone	$+8^{s}$, early January	-8^{s}, early July
obliquity alone	up to $+22^{s}$, solstices	down to -20^{s}, equinoxes

Neither difference is large, which is the point of the film: half a minute a day is invisible, but the days do not average out quickly, and the seconds add up to the sixteen minutes of section 5. The equation of time is the running total of these daily differences.

7. Arc into time

The sky turns 360° in 24^{h}, and the factor is fifteen all the way down.

time into arc	arc into time
$1^{h} = 15^{\circ}$	$1^{\circ} = 4^{m}$
$1^{m} = 15'$	$1' = 4^{s}$
$1^{s} = 15''$	$1'' = \frac{1}{15}{}^{s}$

Examples. The equation of time at its largest, $16^{m}\,27^{s}$, is $4^{\circ}\,06.7'$ of arc. $1^{h}\,20^{m}$ is $15^{\circ} + 5^{\circ} = 20^{\circ}$. One second of time is a quarter of a minute of arc.

8. Longitude from the noon Sun, step by step

Step 1. Greenwich noon. On 3 November 2026 the equation of time is $+16^{\mathrm{m}}\,27^{\mathrm{s}}$, so the Sun crosses the Greenwich meridian at $12^{\mathrm{h}} - 16^{\mathrm{m}}\,27^{\mathrm{s}} = 11{:}43{:}33$ UT. Navigators read this from the tables, to the minute: **11:44**.

Step 2. Your noon. Watch the Sun climb, and note the UTC time at which it stops climbing and stands on your meridian. In the example the watch reads **13:04**.

Step 3. The difference.

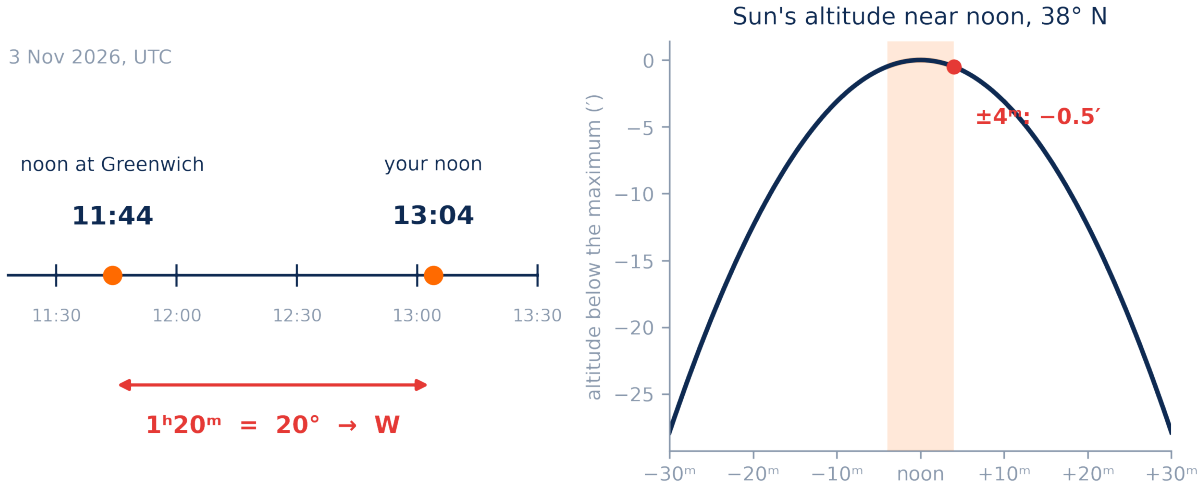
$$13{:}04 - 11{:}44 = 1^{\mathrm{h}}\,20^{\mathrm{m}}.$$

Step 4. Into arc.

$$1^{\mathrm{h}}\,20^{\mathrm{m}} = 15^\circ + 5^\circ = 20^\circ.$$

Step 5. The name. Your noon is later than Greenwich's, so the Sun reached Greenwich first: you are west of it. Later means west, earlier means east — the same rule as $\mathrm{LHA} = \mathrm{GHA} - \lambda_W$.

$$\boxed{\lambda = 20^\circ\ \mathrm{W}}$$



Why the equation of time cancels. Both noons belong to the same true Sun on the same day, so whatever the equation of time is, it is in both times and drops out of the difference. You need it only to know when Greenwich noon happens. In the example the boat sits at exactly 20° W, and its unrounded noon is 13:03:33 — one hour and twenty minutes after Greenwich to the second.

How good is it?

error in the time of noon	error in longitude
1^{s}	$15'' = 0.25'$
1^{m}	$15'$ = 15 NM on the equator, 11.8 NM at 38°
4^{m}	1°

The trouble is judging the moment. At 38° N on 3 November the Sun's declination is 15.2° S and its noon altitude 36.8°. Four minutes either side of noon it is only $0.5'$ lower than at its highest, and ten minutes either side, $3.1'$ — the flat top of the curve in the figure. A sextant cannot pick the top of that curve to

within a minute or two of time. And a moving yacht complicates it further: the highest altitude is not quite the moment of meridian passage, because the boat's own northward or southward run changes the altitude too. So the noon longitude is a check, not a fix.

Which UT? The tables are written in UT1, time kept by the Earth's turning; your watch and GPS show UTC, kept by atomic clocks. Leap seconds hold the two within 0.9^{s} of each other, and

$$0.9^{\mathrm{s}} \times 15''/\mathrm{s} = 13.5'' \approx 0.2',$$

far below anything a noon sight can resolve.

9. Harrison's clock

Section 8 needs Greenwich time at sea, and for most of the history of navigation that was the one thing a ship did not have. John Harrison completed his fourth marine timekeeper, H4, in 1759. It was tried at sea on a voyage to Jamaica in 1761–62, and again to Barbados in 1764, and both times it kept Greenwich time well enough to give longitude. That is the sense in which the film says it took until the 1760s to prove that such a clock could be built.

10. What this document does not cover

Deliberately left out, each being a subject of its own: **the clocks you actually use** — GMT, UT and UTC in detail, zone time and ship's time; **the almanac** — its pages, increments and corrections, including the Moon's; **calendars** — Julian, Gregorian and leap years; and **time signals** and the rate of a chronometer.

Every figure in this document has been computed rather than copied. The equation of time, its two components, the length of the solar day, the meridian passages and the Sun's declination come from the same solar ephemeris used to draw the film, evaluated for 2026; it agrees with the full theory to within a second of time. The worked example in section 8 uses the Greenwich noon from the table in section 5.

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