Where Am I?
Ask the Triangle

The astronomical triangle PZX: you, the sky and the deck

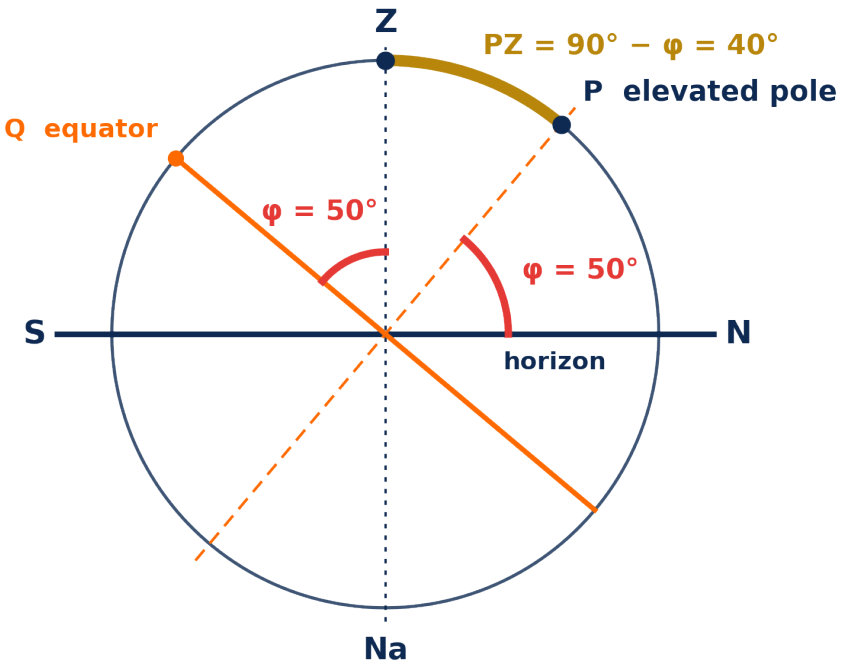
The equatorial system gives every body an address in the sky. The horizon system gives it a second one that belongs to you. This is the companion to Episode 6 — where the two meet, the triangle they make with all six elements labelled, the sign rules, and the two formulas that solve it, worked through on the example from the film.

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1. The two systems meet on your meridian

Your celestial meridian belongs to both systems. In the equatorial system it is the meridian your hour angles start from; in the horizon system it is the principal vertical. It is one great circle, through both celestial poles and through your zenith.



The plane of your meridian, drawn for latitude 50° N. Q is where the celestial equator crosses your meridian above the horizon.

The zenith's declination is your latitude. Your zenith is straight above your position on Earth, so its distance north or south of the celestial equator is your latitude φ.

The pole stands above your horizon at your latitude. The pole is 90° from the equator, the horizon is 90° from the zenith. Turn one frame by φ and the other follows:

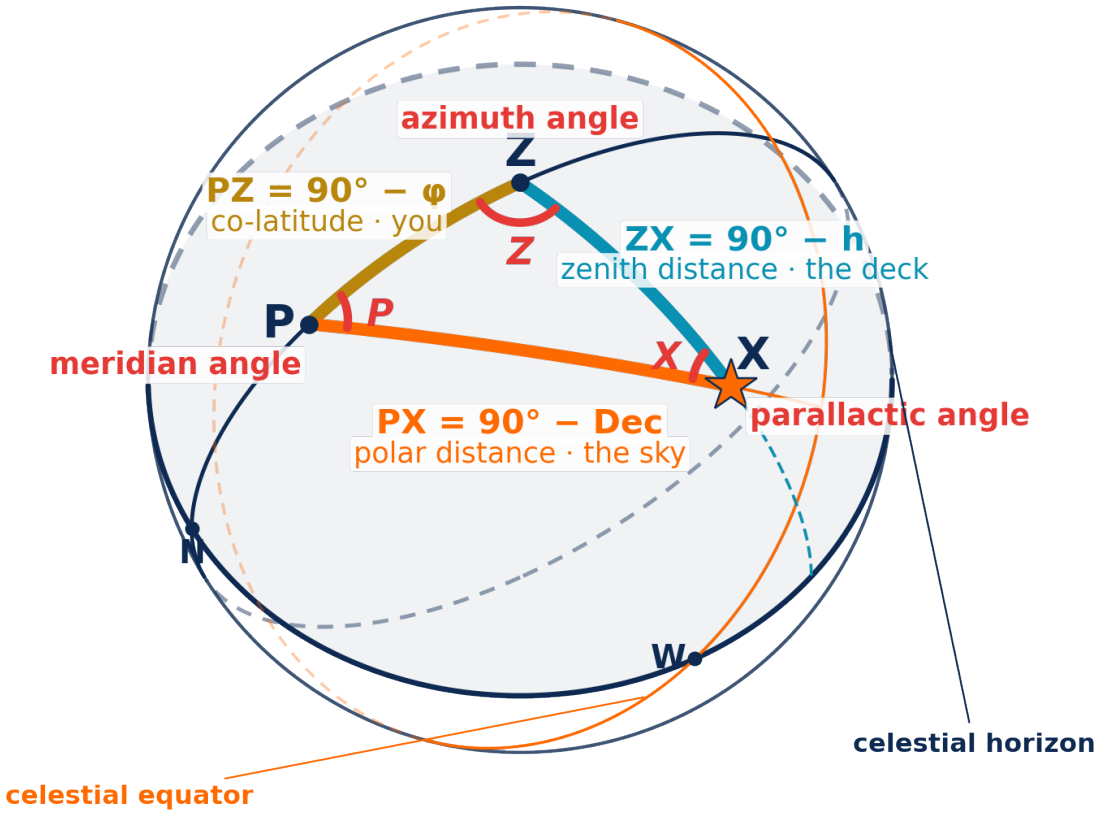
$$\boxed{\text{altitude of the elevated pole} = \varphi}$$

At the equator the pole sits on the horizon; at the North Pole it is overhead. The pole above your horizon is the **elevated pole**. It always has the same name as your latitude: north in the northern hemisphere, south in the southern.

Latitude from Polaris. Polaris is close to the north celestial pole, so its altitude is very nearly your latitude. Not exactly: at the end of 2026 it is 0.63° (37.5′) from the pole and circles it once a day, so the raw altitude can be out by up to about that much. The nautical almanac has a short table for exactly this correction. South of the equator there is no bright pole star; the pole has to be found, for example from the Southern Cross (Episode 5).

2. The astronomical triangle, all six elements

Take three points on the celestial sphere: **P**, the elevated pole; **Z**, your zenith; **X**, the body. Join them with arcs of great circles and you have a spherical triangle: the **astronomical triangle**, or simply **PZX**.



The example from the film: observer at 50° N, 20° W; the body at declination 20° N, Greenwich hour angle 65°, so its local hour angle is 45° and it stands west of the meridian. Seen from above; solid lines are on the near side of the sphere, dashed lines on the far side.

3. The six elements in one table

Every side and every angle of PZX is something from an earlier episode. Nothing new — only joined up. The last column is the example from the film, computed from the same geometry the film was drawn from.

	name	what it is	example
PZ	co-latitude	$90° - \varphi$, along your meridian. *You* — it is where you are.	$90° - 50° = 40°$
PX	polar distance	$90° \mp$ Dec, along the body's hour circle. *The sky* — the declination comes from the almanac.	$90° - 20° = 70°$
ZX	zenith distance	$90° - h$, along the body's vertical circle. *The deck* — you measure h with the sextant.	$90° - 43.6° = 46.4°$
P	meridian angle	At the pole, between your meridian and the body's hour circle. From the LHA, named E or W.	45° W
Z	azimuth angle	At the zenith, between your meridian and the body's vertical circle. From N or S towards E or W.	N 113.5° W (Zn 246.5°)
X	parallactic angle	At the body, between its hour circle and its vertical circle. Not needed; named so the triangle is complete.	38.8°

In the film the example is rounded to $h = 43.6°$ and N 114° W, Zn 246°; to the tenth of a degree it is N 113.5° W and Zn 246.5°, which add up to 360° as they must.

4. The sign rules

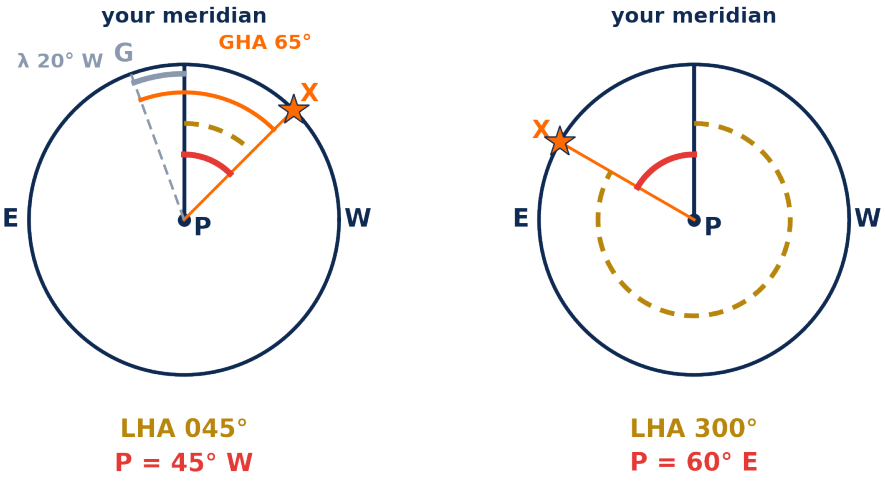
1. The elevated pole has the same name as your latitude. All the rules below count from it.

2. Polar distance: same name or contrary name.

declination and latitude	polar distance	example, at 50° N
same name (both N or both S)	PX = 90° − Dec	Dec N 20°: PX = 70°
contrary name	PX = 90° + Dec	Dec S 15°: PX = 105°

3. Meridian angle from the local hour angle. LHA is counted westward from your meridian, 0° to 360°. The triangle cannot hold an angle that large:

body west (LHA $< 180°$) : $P = \text{LHA W}$ body east (LHA $> 180°$) : $P = 360° - \text{LHA E}$



Seen from above the north celestial pole. Left: the example from the film, with the Greenwich meridian G. Right: a body east of the meridian.

4. Local hour angle from Greenwich: this is where your longitude hides.

$$\text{LHA} = \text{GHA} + \lambda_{\text{E}} \qquad \text{LHA} = \text{GHA} - \lambda_{\text{W}}$$

Add or subtract 360° to bring the result between 0° and 360°. In the example: $65° - 20° = 45°$.

5. Azimuth angle and true azimuth. The azimuth angle Z is counted from N or from S — the side of the elevated pole — towards E or W, never more than 180°. The true azimuth Zn is counted from north, clockwise, all the way round:

azimuth angle	true azimuth	example
N Z E	$\text{Zn} = Z$	N 60° E = 060°
N Z W	$\text{Zn} = 360° - Z$	N 113.5° W = 246.5°
S Z E	$\text{Zn} = 180° - Z$	S 60° E = 120°
S Z W	$\text{Zn} = 180° + Z$	S 60° W = 240°

5. The two formulas that solve it

On deck you measure the altitude h (sextant) and the azimuth Zn (compass), and note the moment. From the almanac, for that moment, you take the declination and the Greenwich hour angle. In the triangle that is two sides and the angle at the zenith:

$$h \;\to\; \mathrm{ZX} = 90^\circ - h \qquad \mathrm{Zn} \;\to\; Z \qquad \mathrm{Dec} \;\to\; \mathrm{PX} = 90^\circ - \mathrm{Dec}$$

Formula 1 — the cosine rule for the side PX gives the latitude.

$$\boxed{\sin \mathrm{Dec} = \sin\varphi \sin h + \cos\varphi \cos h \cos Z}$$

Signs: take φ as a positive number, Dec positive when it has the same name as your latitude and negative when contrary (this is rule 2 again), and Z counted from the elevated pole.

The unknown φ appears twice. Write $a = \sin h$, $b = \cos h \cos Z$, $c = \sin \mathrm{Dec}$; then

$$a\sin\varphi + b\cos\varphi = c \quad \Rightarrow \quad \varphi = \theta \pm \arccos\frac{c}{R}, \qquad R = \sqrt{a^2+b^2}, \quad \cos\theta = \frac{b}{R},\ \sin\theta = \frac{a}{R}.$$

Of the two roots, keep the one that is a latitude (between 0° and 90°), and the one close to your dead-reckoning position if both are.

Formula 2 — the sine rule gives the angle at the pole.

$$\boxed{\sin P = \frac{\sin Z \cos h}{\cos \mathrm{Dec}}}$$

A sine does not tell P from $180^\circ - P$. When in doubt, the cosine rule for the side ZX settles it: $\cos P = (\sin h - \sin\varphi \sin \mathrm{Dec})/(\cos\varphi \cos \mathrm{Dec})$. Then rule 3 turns P into the LHA, and rule 4 turns the LHA into your longitude: $\lambda = \mathrm{LHA} - \mathrm{GHA}$, east if positive, west if negative.

6. Worked example: the film's triangle

on deck		**from the almanac**	
altitude	$h = 43^\circ 33.6'$	declination	$\mathrm{Dec} = \mathrm{N}\ 20^\circ 00.0'$
azimuth	$\mathrm{Zn} = 246^\circ 28.9'$, so $Z = \mathrm{N}\ 113^\circ 31.1'\ \mathrm{W}$	Greenwich hour angle	$\mathrm{GHA} = 65^\circ 00.0'$

Latitude. $a = \sin h = 0.6891$, $b = \cos h \cos Z = -0.2892$, $c = \sin \mathrm{Dec} = 0.3420$.

$$R = 0.7473, \quad \theta = 112.76^\circ, \quad \arccos\frac{c}{R} = 62.76^\circ \quad \Rightarrow \quad \varphi = 50.00^\circ \text{ or } 175.53^\circ$$

Only the first is a latitude: $\boxed{\varphi = 50^\circ 00.0'\ \mathrm{N}}$, and $\mathrm{PZ} = 40^\circ$.

Angle at the pole. $\sin P = \sin 113.52^\circ \cdot \cos 43.56^\circ / \cos 20^\circ$, so $P = 45^\circ 00.0'$. Check: $\cos P = 0.7071$, also 45°, not 135°. The body is west of you (Zn between 180° and 360°), so P is west and $\mathrm{LHA} = 45^\circ$.

Longitude. $\lambda = \mathrm{LHA} - \mathrm{GHA} = 45^\circ - 65^\circ = -20^\circ$: $\boxed{\lambda = 20^\circ 00.0'\ \mathrm{W}}$.

Latitude and longitude: a position on the chart. In theory.

7. If only it were that simple

The altitude. The angle your sextant reads is not yet the altitude h in the triangle. Why, and what it takes to make it one, is the next episode.

The azimuth. Formula 1 is sensitive to Z. In the example, an error of 1° in the compass bearing moves the latitude by about $38'$ to $39'$ — some 38 nautical miles — and the longitude by about $26'$, some 16 to 17 nautical miles at this latitude. That is why navigators do not take their position from a compass bearing.

The method. The navigators who did this every day almost never solved the triangle this way. How they did, comes in the next episodes.

Try it. On the next clear night, measure — or just estimate — the altitude of Polaris, and tell us in the comments how close it came to your latitude.

Every number in this document has been computed rather than copied. The figures and the worked example are drawn from the same geometry as the film (observer 50° N, 20° W; body Dec N 20°, GHA 65°), and both formulas have been checked against it. Polaris' distance from the pole is its J2000 position carried forward to the end of 2026 by the standard precession formulae, as in the Episode 5 PDF.

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